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B028415(028)

B. Tech. (Fourth Semester) Examination, April-May 2022

(AICTE Scheme)

(Electronics and Telecommunication Engg. Branch)

PROBABILITY THEORY and STOCHASTIC PROCESSES

Time Allowed : Three hours

Maximum Marks : 100

Minimum Pass Marks : ~~40~~ 35

Note : *Attempt all questions. Every question has four parts. Part (a) is compulsory from each unit. Attempt any **two** parts from (b), (c) and (d).*

Unit-I

1. (a) State and prove the Baye's theorem. 4

(b) An airline in the small city has 5 departures each day. It is known that any given flight has a probability of 0.3 of departing late. For any given day find the probabilities that :

(i) no flights depart late

(ii) all flights depart late

(iii) three or more depart on time 8

(c) A company sells high fidelity amplifiers capable of generating 10, 25 and 50 W audio power. It has on hand 100 of the 10 W units, of which 15% are defective, 70 of the 25 W units with 10% defective and 30 of the 50 W units with 10% defective.

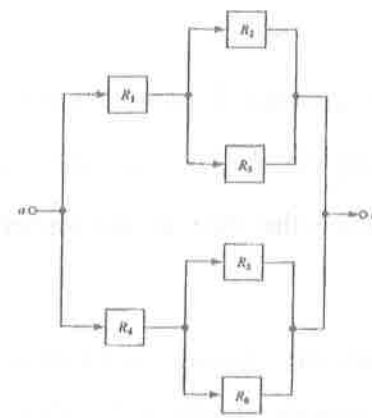
(i) What is the probability that an amplifier sold from the 10 W units is defective?

(ii) If each wattage amplifier sells with equal likelihood, what is the probability of a randomly selected unit being 50 W and defective.

(iii) What is the probability that a unit randomly selected for sale is defective? 8

(d) In a communication system the signal send from point a to point b arrives by two paths as shown in

figure. All repeaters fail independently of each other. The probability of failing of repeaters are P (R1) = 0.005, P (R2) = P (R3) = P (R4) = 0.01 and P (R5) = P (R6) = 0.05. Find the probability that the signal will not arrive at point b. 8



Unit-II

2. (a) Define random variable. Explain the difference between discrete and continuous random variable taking suitable examples. 4

(b) Consider an experiment of "Rolling of two dice". Take the random variable as "the sum of two numbers that show on the dice." Find the plot the corresponding cumulative distribution function and probability density function. 8

(c) The pdf is given as $f_x(X) = ae^{-b|x|}$, where X is a random variable whose allowable values range from $-\infty$ and ∞. Find :

(i) The relationship between a and b

(ii) Find the plot the corresponding CDF and pdf

(iii) Probability that the outcome lies between 1 and 2. 8

(d) The lifetime of a system expressed in weeks is a Rayleigh random variable X for which :

$$f_x(x) = \begin{cases} \left(\dfrac{x}{200}\right)\exp\left(-\dfrac{x^2}{400}\right) & \text{for} \quad x \geq 0 \\ 0 & \text{for} \quad x < 0 \end{cases}$$

(i) What is the probability that the system will not last a full week?

(ii) What is the probability that the system lifetime will exceed one year? 8

Unit-III

3. (a) Explain the probability density of sum of random variables. Write the central limit theorem. 4

(b) Find the mean and variance of a random variable X which is uniformly distributed between a and b. Assume that $b > a$. 8

(c) The standard exponential pdf is given as :

$$f_x(x) = \begin{cases} \left(\dfrac{1}{b}\right)e^{-(x-a)/b} & x \geq a \\ 0 & x < \alpha \end{cases}$$

where $b > 0$ and $-\infty < a < +\infty$.

Determine the mean, mean square value and variance for exponential pdf. 8

(d) A random variable X has $E[X] = 3$, $E\left[X^2\right] = 11$ and $Var[X] = 2$. For a new random variable $Y = 2X - 3$, find $E[Y]$, $E\left[Y^2\right]$ and $Var[Y]$. 8

Unit-IV

4. (a) Define random process. Classify random processes according to their characteristics. 4

(b) Let X (t) be a wide sense stationary random process with autocorrection function

$R_{XX}(\tau) = e^{-a|\tau|}$ where $a > 0$ is constant.

If X(t) "amplitude modulates" a "carrier" $\cos(w_0 t + \phi)$ where w_0 is constant and ϕ is a random variable which is uniformly distributed in the range of $(-\pi, \pi)$ and statistically independent of $X(t)$. Determine the autocorrelation function of $Y(t) = X(t)\cos(w_0 t + \phi)$. 8

(c) Let two random processes $X(t)$ and $Y(t)$ be defined as :

$$X(t) = A\cos(w_o t) + B\sin(w_o t)$$

$$Y(t) = B\cos(w_o t) - A\sin(w_o t)$$

Where A and B are random variables and w_0 is a constant $X(t)$ is wide sense stationary since A and B are uncorrelated, zero mean random variables with the same variances. With the same constants on A and B, $Y(t)$ is also wide-sense stationary. Show that $X(t)$ and $Y(t)$ are jointly wide-sense stationary. 8

(d) A zero mean wide sense stationary process $X(t)$ has an autocorrelation function

$R_{XX}(\tau) = C_{XX}(\tau) = \exp(-2\alpha|\tau|)$ for $\alpha > a$ a constant

Determine if $X(t)$ is a mean exgodic process. 8

Unit-V

5. (a) Define power density spectrum. Write its properties. 4

(b) A wide sense stationary process $X(t)$ has an automorphism function :

$$R_{XX}(\tau) = \left\{ A_0\left[1 - \left(|\tau|/T\right)\right] \text{for} T <= t <= T \text{and} \right.$$

$$\left. 0 \text{ elsewhere} \right\}$$

where $t > 0$ and A_0 is a constant. Determine the power spectrum. 8

(c) Consider the random process $X(t) = A_o \cos(w_0 t + \phi)$ where A_o and w_o real constants and ϕ is random variable which is uniformly distributed in the range of $(0, \pi/2)$. Find the aveage power P_{XX} in $X(t)$. 8

(d) What do you understand by white and coloured noise in terms of power spectrum? Also explain the response of product device to a random signal like noise. 8